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Light Scattering at the Uniaxial-Biaxial Transition in Nematic Liquid Crystals

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Light Scattering at the Uniaxial-Biaxial Transition in Nematic Liquid Crystals

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An order parameter is proposed for the uniaxial-biaxial transition in nematic-liquid crystals. The Landau-Ginzburg part of the free energy and the linearized equations of motion for the order parameter and hydrodynamic variables in the uniaxial phase are then derived. The equations are used to predict the power spectrum of light scattering above the uniaxial-biaxial transition.

1. INTRODUCTION

Recently, the existence of a biaxial nematic liquid crystal was experimentally verified by Yu and Saupe[1] in potassium laurate (KL)-1-decanol-D_2O mixtures. This new biaxial phase is characterized by long range orientational order in two (hence all three) of the spatial directions. A biaxial nematic has rotational symmetry broken around three distinct axes.

In a limited KL concentration range the following sequence of phase transitions was observed reversibly upon heating and cooling: A first order transition from the isotropic fluid to the uniaxial nematic phase, followed by a second order transition from the uniaxial nematic to the biaxial nematic phase. In Section 2 of this paper we propose an order parameter for the uniaxial-biaxial (u-b) transition in nematics and calculate the form of the order parameter static correlation function. In Section 3 we calculate the linearized equations of motion for the order parameter and the hydrodynamic variables of the uniaxial phase just above the u-b transition. In Section 4 we predict the form of the spectrum for light scattering near the u-b transition using two different scattering geometries. A summary and conclusion follow in Section 5.

2. THE ORDER PARAMETER

The orientational order in a biaxial nematic liquid crystal can be described by the symmetric, traceless, second rank tensor order parameter[2]

$$Q_{ij}(\mathbf{r}) = S(\mathbf{r})[n_i(\mathbf{r})n_j(\mathbf{r}) - \tfrac{1}{3}\delta_{ij}] + \xi(\mathbf{r})[m_i(\mathbf{r})m_j(\mathbf{r}) - l_i(\mathbf{r})l_j(\mathbf{r})], \quad (1)$$

where $S(\mathbf{r})$ is a measure of the uniaxiality and $\xi(\mathbf{r})$ is a measure of the biaxiality in $Q_{ij}(\mathbf{r})$. The "uniaxial director" $n_i(\mathbf{r})$ and the "biaxial director" $m_i(\mathbf{r})$ are orthogonal unit vectors defining the local symmetry axes and $l_i(\mathbf{r}) = \epsilon_{ijk}m_jn_k$ where ϵ_{ijk} is the totally antisymmetric tensor.

In the principal axes system this order parameter has three distinct eigenvalues, and is appropriate for describing biaxial nematics with orthorhombic symmetry.† We have made extensive use of this order parameter in deriving the hydrodynamic equations for the biaxial phase in nematic liquid crystals.[3] For the purposes of this paper we define n_i^0, the equilibrium direction of n_i, to be in the z-direction, and m_i^0 to be in the y-direction.

Since the order parameter Q_{ij} contains information about both the uniaxial order $[S(\mathbf{r})]$ and the biaxial order $[\xi(\mathbf{r})]$, it cannot be used as an order parameter for the uniaxial-biaxial transition. An appropriate order parameter for this transition can be constructed by extracting the anisotropic part of Q_{ij} in the plane perpendicular to the uniaxial axis. We define

$$\tilde{\xi}_{ij} = Q^{\perp}_{ij} - \tfrac{1}{2}\delta^{\perp}_{ij}Q^{\perp}_{kk} \quad (2)$$

where

$$\delta^{\perp}_{ij} = \delta_{ij} - n_in_j \quad (3)$$

is the projection operator onto the plane perpendicular to n_i, and

$$Q^{\perp}_{ij} = \delta^{\perp}_{ik}Q_{kl}\delta^{\perp}_{lj}.$$

An equally valid order parameter for the uniaxial-biaxial transition can be found by extracting the anisotropic part, in the plane perpen-

† Liu[4] has recently argued for a relaxation of this restriction and considered biaxial nematics with other discrete symmetries.

dicular to the uniaxial axis, of any macroscopic tensor property of the ordered phase. In particular, we could, in analogy with de Gennes,[5] define

$$\xi_{ij} = \chi^{\perp}_{ij} - \tfrac{1}{2}\,\delta^{\perp}_{ij}\chi^{\perp}_{kk} \qquad (4)$$

where χ_{ij} is the magnetic susceptibility tensor. In either case, the order parameter for the uniaxial-biaxial transition in nematics is seen to be a symmetric, traceless, second rank tensor with two independent components corresponding to a magnitude, and a direction for the principal axis in the plane perpendicular to n^0_i. Explicitly, we write

$$\xi_{ij} = \begin{pmatrix} \xi_{xx} & \xi_{xy} & 0 \\ \xi_{xy} & -\xi_{xx} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (5)$$

The Landau-Ginzburg portion of the free energy is an expansion in scalar invariants of the order parameter and its gradients:

$$F_\xi = \frac{1}{2}\int d^3x\{a\xi_{ij}\xi_{ij} + C_{\parallel}(n_k\nabla_k\xi_{ij})(n_l\nabla_l\xi_{ij})$$
$$+ C_{\perp}\delta^{\perp}_{kl}(\nabla_k\xi_{ij})(\nabla_l\xi_{ij})\} + 0(\xi^4), \qquad (6)$$

where a $\alpha(T - T_{u\text{-}b})$ and $C_{\parallel}$ are $C_{\perp}$ are constants analogous to those introduced in Chen and Lubensky.[6] The only fundamental invariant of the order parameter is

$$\xi_{ij}\xi_{ij} = 2(\xi^2_{xx} + \xi^2_{xy}). \qquad (7)$$

The absence of a cubic term in the free energy implies that the uniaxial-biaxial transition is second order, in agreement with both experiment,[1] and previous mean field type calculations.[7,8,9]

The order parameter-order parameter static correlation function is readily calculated by taking the Fourier transform of Eq. (6),

$$\langle\xi_{ij}(\mathbf{k})\xi_{kl}(\mathbf{k}')\rangle = (2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}')\{\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\}k_BT\chi_0(\mathbf{k}), \qquad (8)$$

where the inverse susceptibility is given by

$$\chi_0^{-1}(\mathbf{k}) = a + C_{\parallel}k^2_{\parallel} + C_{\perp}k^2_{\perp}, \qquad (9)$$

and

$$k^2_{\perp} = k^2_x + k^2_y.$$

In real space, the correlation function is an anisotropic Ornstein-

Zernike form: For correlations parallel to the uniaxial axis

$$\chi_0(\mathbf{r}) \sim \frac{1}{r} e^{-\kappa_\parallel r}, \tag{10}$$

where the inverse correlation length $\kappa_\parallel = (a/C_\parallel)^{1/2}$. For correlations in the plane perpendicular to the uniaxial axis

$$\chi_0(\mathbf{r}) \sim \frac{1}{r} e^{-\kappa_\perp r} \tag{11}$$

where the inverse correlation length $\kappa_\perp = (a/C_\perp)^{1/2}$.

3. THE EQUATIONS OF MOTION

In the uniaxial phase near the uniaxial-biaxial transition the order parameter is a slowly varying quantity and its equation of motion couples to the set of hydrodynamic equations. The hydrodynamic variables in the uniaxial nematic liquid are those due to microscopic conservation laws and those arising from the continuous broken rotational symmetry of the phase.[10] The densities of the conserved quantities are the mass density ρ, the energy density ϵ, and the momentum density $g_i = v_i(\rho_0 = 1)$, where v_i is the velocity. The variables corresponding to the continuous broken rotational symmetry are the fluctuating components of the director, δn_x and δn_y. The linearized equations of motion for these hydrodynamic variables can be found in Forster, *et al.*[10] We have coupled the order parameter to the hydrodynamic variables using the entropy production formalism.†

The equation of motion for the order parameter can be written

$$\dot{\xi}_{ij} = -\phi_{ij} \tag{12}$$

where ϕ_{ij} is a flux. Since ξ_{ij} is even under time reversal the reactive part of ϕ_{ij}, ϕ_{ij}^R, must be odd. The only hydrodynamic variable which is odd in time is the velocity v_i; therefore, ϕ_{ij}^R must be an odd function of v_i or its spatial derivatives.

If we write the equation of motion for the velocity as

$$\dot{v}_i = -\nabla_i p - \nabla_j \sigma_{ij} \tag{13}$$

where p is the pressure and σ_{ij} is the symmetric stress tensor, then the

† See for example Martin, Parodi, and Pershan[11] or Miyano and Ketterson.[12]

reactive contributions of the order parameter and the velocity to the entropy production term, after taking the Fourier transform of the spatial coordinate, is

$$\dot{S} = \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{\phi_{ij}^R(\mathbf{k})}{T} \chi_0^{-1}(-\mathbf{k})\xi_{ij}(-\mathbf{k}) - \frac{\sigma_{ij}^R(\mathbf{k})}{T} A_{ij}(-\mathbf{k}) \right\} \tag{14}$$

where

$$A_{ij}(\mathbf{k}) = \frac{i}{2}\left[k_j v_i(\mathbf{k}) + k_i v_j(\mathbf{k})\right] \tag{15}$$

and T is the temperature. Since reactive terms cannot contribute to entropy production we find

$$\sigma_{ij}^R(\mathbf{k}) = -\tilde{\tau}_{ijkl}\chi_0^{-1}(\mathbf{k})\xi_{kl}(\mathbf{k}) \tag{16}$$

and

$$\phi_{kl}^R(\mathbf{k}) = \tau_{ijkl}A_{ij}(\mathbf{k}), \tag{17}$$

where τ_{ijkl} and $\tilde{\tau}_{ijkl}$ are reactive coefficients related by the Onsager reciprocity relation;[13]

$$\tilde{\tau}_{ijkl} = -\tau_{ijkl}. \tag{18}$$

The explicit form of τ_{ijkl} can be calculated by general invariance arguments. It must be consistent with both the underlying uniaxial symmetry of the phase, and the explicit symmetries of ξ_{kl} and A_{kl}. The only admissable form is

$$\tau_{ijkl} = \frac{\tau}{2}\left(\delta_{ik}^{\perp}\delta_{jl}^{\perp} + \delta_{il}^{\perp}\delta_{jk}^{\perp} - \delta_{ij}^{\perp}\delta_{kl}^{\perp}\right). \tag{19}$$

Irreversible effects enter the equations of motion with opposite time-reversal properties to reactive effects. The dissipative part of ϕ_{ij}, ϕ_{ij}^D, must be proportional to variables which are even in time: ρ, ϵ, δn_i, and ξ_{ij}. Since the mass current is conserved it cannot be dissipatively coupled to ϕ_{ij}^D. The director also behaves like a conserved quantity, so we can ignore its coupling to ϕ_{ij}^D as well. The contribution of the dissipative terms to the entropy production is then,

$$\dot{S} = \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{\phi_{ij}^D(\mathbf{k})}{T} \chi_0^{-1}(-\mathbf{k})\xi_{ij}(-\mathbf{k}) + i\,\frac{j_i^{\epsilon}(\mathbf{k})^D}{T^2} k_i T(-\mathbf{k}) \right\} \tag{20}$$

where

$$\dot{\epsilon} = -\nabla_i j_i^{\epsilon}. \tag{21}$$

Cross terms between the order parameter and the energy density are higher order in **k** than we will consider. We find

$$\phi_{ij}^{D}(\mathbf{k}) = \zeta_{ijkl}\chi_0^{-1}(-\mathbf{k})\xi_{kl}(-\mathbf{k}), \tag{22}$$

where ζ_{ijkl} are relaxation coefficients.

Invariance arguments again require that

$$\zeta_{ijkl} = \frac{\zeta}{2}(\delta_{ik}^{\perp}\delta_{jl}^{\perp} + \delta_{il}^{\perp}\delta_{jk}^{\perp} - \delta_{ij}^{\perp}\delta_{kl}^{\perp}), \tag{23}$$

so that the linearized equations of motion in the uniaxial phase just above the u-b transition consist of the equations for ρ, ϵ, and δn_i; plus

$$\dot{\xi}_{ij}(\mathbf{k}, t) = -\zeta\chi_0^{-1}(\mathbf{k})\xi_{ij}(\mathbf{k}, t) - \tau\tilde{A}_{ij}(\mathbf{k}, t) \tag{24}$$

where $\tilde{A}_{ij} = A_{ij}^{\perp} - \frac{1}{2}\delta_{ij}^{\perp}A_{kk}^{\perp}$, and

$$\dot{v}_i(\mathbf{k}, t) = -ik_i p(\mathbf{k}, t) + i\eta_{ijkl}k_j A_{kl}(\mathbf{k}, t) - i\tau\chi_0^{-1}(\mathbf{k})k_j\xi_{ij}(\mathbf{k}, t) + (\text{other terms}) \tag{25}$$

where η_{ijkl} is the familiar matrix of viscosity coefficients for the uniaxial phase. For an incompressible fluid ($\nabla_i v_i = 0$),

$$\eta_{ijkl} = +\nu_2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + (\nu_3 - \nu_2)(\delta_{ik}n_l^0 n_j^0 + \delta_{jk}n_l^0 n_i^0 + \delta_{il}n_k^0 n_j^0 + \delta_{jl}n_k^0 n_i^0) + 2(\nu_1 + \nu_2 - 2\nu_3)n_i^0 n_j^0 n_k^0 n_l^0. \tag{26}$$

For very low frequency and wave vector these equations reduce to usual hydrodynamic ones in the uniaxial phase. The eigenvalues are the same as those reported in Forster, *et al.*[10]

4. LIGHT SCATTERING

Light scattering in a nematic liquid is caused by fluctuations in the dielectric tensor, $\delta\epsilon_{kl}$. In the uniaxial nematic phase it has been shown[14] that the light scattering is dominated by fluctuations in the orientation of the principal axis, so that

$$\delta\epsilon_{kl} \propto \tfrac{1}{2}(n_k^0\delta n_l + n_l^0\delta n_k) \equiv N_{kl} \tag{27}$$

Near the phase transition, the dielectric tensor is also proportional to the order parameter ξ_{kl}. Since the time dependent spectrum of light scattering $I(\mathbf{k}, t)$, at a given wave vector **k**, is given by[15] (if a time argument is not explicitly indicated, time zero is meant)

$$I(\mathbf{k}, t) \propto \langle (\hat{i}_k \delta\epsilon_{kl}(\mathbf{k}, t)\hat{s}_l)(\hat{i}_{k'} \delta\epsilon_{k'l'}(-\mathbf{k})\hat{s}_{l'})\rangle \tag{28}$$

where $\hat{i}_k$ and $\hat{s}_l$ are the polarization vectors of the incident and scattered radiation, respectively; we can write

$$I(\mathbf{k}, t) \propto \langle (\hat{i}_k N_{kl}(\mathbf{k}, t)\hat{s}_l)(\hat{i}_{k'} N_{k'l'}(-\mathbf{k})\hat{s}_{l'})\rangle + \langle (\hat{i}_k \xi_{kl}(\mathbf{k}, t)\hat{s}_l)(\hat{i}_{k'} \xi_{k'l'}(-\mathbf{k})\hat{s}_{l'})\rangle + (\text{cross terms}). \tag{29}$$

The contribution of the first term to the light scattering spectrum has been studied by de Gennes.[14] We will calculate, to linear order, the contribution due to the order parameter for the u-b transition and discuss the possibility of pretransitional fluctuations affecting the first term. To linear order there is no coupling of the order parameter to the director so that the cross terms vanish. Two different scattering geometries, chosen to exploit the anisotropy of the uniaxial phase will be considered. One results in a wave vector **k** which is parallel to the uniaxial axis, the other has **k** lying in the plane perpendicular to the uniaxial axis. The results are two different spectral line shapes.

If we choose the x-axis as the direction of incident polarization, the y–z plane as the scattering plane, define the angle between the incident direction and the y-axis to be $\theta/2$ and the angle between the scattered direction and the y-axis to be $-\theta/2$, then **k** is in the z-direction and the polarized scattering spectrum is given by

$$I_{vv}(\mathbf{k}, t) \propto \langle \xi_{xx}(\mathbf{k}, t)\xi_{xx}(-\mathbf{k})\rangle \tag{30}$$

The depolarized spectrum for this scattering geometry is

$$I_{vh}(\mathbf{k}, t) \propto \langle \delta n_x(\mathbf{k}, t)\delta n_x(-\mathbf{k})\rangle \cos^2\frac{\theta}{2} + \langle \xi_{xy}(\mathbf{k}, t)\xi_{xy}(-\mathbf{k})\rangle \sin^2\frac{\theta}{2}. \tag{31}$$

With **k** in the z-direction the equations of motion for the independent components of the order parameter are

$$[\partial_t + \zeta\chi_0^{-1}(\mathbf{k})]\xi_{xx}(\mathbf{k}, t) = 0 \tag{32}$$

and

$$[\partial_t + \zeta\chi_0^{-1}(\mathbf{k})]\xi_{xy}(\mathbf{k}, t) = 0. \tag{33}$$

This particular choice of scattering geometry decouples the velocity and order parameter equations of motion. Multiplying both sides of Eq. (32) by $\xi_{xx}(-\mathbf{k})$, averaging, then taking the Laplace transform yields

$$[s + \zeta\chi_0^{-1}(\mathbf{k})]L_s\{\langle\xi_{xx}(\mathbf{k}, t)\xi_{xx}(-\mathbf{k})\rangle\} = \chi_0(\mathbf{k}), \tag{34}$$

where L_s means the Laplace transform. Since,

$$I_{vv}(\mathbf{k}, \omega) \;\alpha\; 2ReL_s\{\langle\xi_{xx}(\mathbf{k}, t)\xi_{xx}(-\mathbf{k})\rangle\}_{s=-i\omega} \tag{35}$$

we find

$$I_{vv}(\mathbf{k}, \omega) \;\alpha\; \frac{2\Gamma(\mathbf{k})\chi_0(\mathbf{k})}{\omega^2 + \Gamma^2(\mathbf{k})}, \tag{36}$$

where

$$\Gamma(\mathbf{k}) = \zeta\chi_0^{-1}(\mathbf{k}) = \zeta(a + C_\parallel k_\parallel^2). \tag{37}$$

The power spectrum for this scattering geometry is a simple Lorentzian with half-width $\Gamma(\mathbf{k})$ reflecting the purely relaxational behavior of $\xi_{xx}(\mathbf{k}, t)$.

The equation of motion for $\xi_{xy}(\mathbf{k}, t)$ is also decoupled in this geometry, so its contribution to the depolarized spectrum is a simple Lorentzian as well;

$$I_{vh}(\mathbf{k}, \omega) \;\alpha\; 2ReL_s\{\delta n_x(\mathbf{k}, t)\delta n_x(-\mathbf{k})\}_{s=-i\omega} \cos^2\frac{\theta}{2} + \frac{2\Gamma(\mathbf{k})\chi_0(\mathbf{k})}{\omega^2 + \Gamma^2(\mathbf{k})}\sin^2\frac{\theta}{2}. \tag{38}$$

An alternative scattering geometry is one in which $\mathbf{k}$ lies entirely in the plane perpendicular to the uniaxial axis. The scattering plane is the y–z plane. The angle between the incident direction and the z-axis is $\phi/2$. This results in a wave vector $\mathbf{k}$ aligned along the y-axis. If the direction of incident polarization is chosen to be in the x direction, then the polarized spectrum is

$$I_{vv}(\mathbf{k}, t) \;\alpha\; \langle\xi_{xx}(\mathbf{k}, t)\xi_{xx}(-\mathbf{k})\rangle, \tag{39}$$

while the depolarized spectrum is

$$I_{vh}(\mathbf{k}, t) \;\alpha\; \langle\delta n_x(\mathbf{k}, t)\delta n_x(-\mathbf{k})\rangle \sin^2\frac{\phi}{2} + \langle\xi_{xy}(\mathbf{k}^-, t)\xi_{xy}(\mathbf{k}^-)\rangle \cos^2\frac{\phi}{2}. \tag{40}$$

Only the depolarized spectrum contains a piece which presents a new feature.

The equation of motion for $\xi_{xy}(\mathbf{k}, t)$ with $\mathbf{k}$ along the y-axis is

$$[\partial_t + \zeta\chi_0^{-1}(\mathbf{k})]\xi_{xy}(\mathbf{k}, t) = -i\tau k_y v_x(\mathbf{k}, t)/2, \tag{41}$$

while the equation of motion for the x-component of the velocity is

$$[\partial_t + \nu_2 k_y^2] v_x(\mathbf{k}, t) = -\, i\tau k_y \chi_0^{-1}(\mathbf{k})\xi_{xy}(\mathbf{k}, t) + \text{(other terms)}. \tag{42}$$

Multiplying Eq. (41) by $\xi_{xy}(-\mathbf{k})$, averaging, then taking the Laplace transform yields

$$[s + \zeta\chi_0^{-1}(\mathbf{k}) + \Theta(\mathbf{k}, s)]L_s\{\langle\xi_{xy}(\mathbf{k}, t)\xi_{xy}(-\mathbf{k})\rangle\} = \chi_0(\mathbf{k}) \tag{43}$$

where

$$\Theta(\mathbf{k}, s) = \frac{\frac{1}{2}\,\tau^2 k_y^2 \chi_0^{-1}(\mathbf{k})}{s + \nu_2 k_y^2} \tag{44}$$

results from the non-zero average of $\langle v_x(\mathbf{k}, t)\ \xi_{xy}(-\mathbf{k})\rangle$. We have used Eq. (42).

The polarized power spectrum is then

$$I_{vv}(\mathbf{k}, \omega)\ \alpha\ 2Re\left[\frac{\chi_0(\mathbf{k})}{-i\omega + \Gamma(\mathbf{k}) + \Theta(\mathbf{k}, \omega)}\right] \tag{45}$$

which can be written

$$I_{vv}(\mathbf{k}, \omega)\ \alpha\ \frac{\omega^2 + \nu_2 A k_y^4}{(2\zeta)^{-1}[(\omega^2 - \Gamma(\mathbf{k})Ak_y^2)^2 + \omega^2(\Gamma(\mathbf{k}) + \nu_2 k_y^2)^2]} \tag{46}$$

where

$$A = \nu_2 + \frac{\tau^2}{2\zeta}. \tag{47}$$

This spectrum has the feature of a central dip at the top of a Lorentzian in the region where the order parameter relaxation rate $\Gamma(\mathbf{k})$ and the velocity relaxation rate $\nu_2 k_y^2$ are of the same order of magnitude.† We would expect this to occur far from the transition point, for as the transition is approached the so-called critical slowing down decreases $\Gamma(\mathbf{k})$

† This line shape was predicted by de Gennes[5] for the isotropic-nematic transition and further studied by Chen and Huang.[17]

drastically. When $\nu_2 k_y^2 \gg \Gamma(\mathbf{k})$ the central dip disappears and we recover the Lorentzian line shape. Of course, if the temperature is so close to $T_{u\text{-}b}$ that $\kappa \ll |\mathbf{k}|$, then the relaxation rate $\Gamma(\mathbf{k})$ is truly $\mathbf{k}$-dependent and the linear theory breaks down. The non-linear effects must be investigated via the mode-coupling theory.[16]

The effect of the director terms on the power spectrum in Eqs. (31) and (39) can be minimized by appropriate choices of the scattering angle. There remains the possibility, however, that pretransitional fluctuations of the order parameter could lead to divergences in the elastic constants hence modifying the power spectrum.[6,18] We have carried out a perturbation expansion in director-order parameter couplings of the free energy (6) in complete analogy with the calculation of Chen and Lubensky.[6] We predict no divergences in the elastic constants of the uniaxial phase as the u-b transition is approached.

This is in accord with our previous calculation of the elastic theory for the biaxial phase showing that bend, splay and twist deformations have vanishing rather than finite energy at zero wave number.

5. CONCLUSION

We have proposed an order parameter for the u-b transition in nematic liquid crystals and calculated the order parameter-order parameter static correlation function. It was found to be of the anisotropic Ornstein-Zernike form. The linearized dynamics in the uniaxial phase just above the transition was then calculated via the entropy production formalism. We predict a reactive coupling between the velocity and the order parameter.

The power spectrum of light scattering near the phase transition was then calculated. Two different scattering geometries were analyzed in order to exploit the uniaxial symmetry of the high temperature phase. One geometry results in a Lorentzian spectrum, providing a measure of the relaxation of pretransitional fluctuations of the order parameter. The other yields a spectrum which shows a central dip at the top of a Lorentzian-type structure. The central dip is a reflection of the coupling of the order parameter and the velocity gradient.

We also report the result of a perturbative calculation on the effect of pretransitional fluctuations of the order parameter on the elastic constants in the uniaxial phase. We find no divergences in these quantities as the u-b transition is approached.

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